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B.A./B.Sc. VI Semester Examination

$NON - CBCS - VIs(P)$

Mathematics
Course No.: MA-601

Time Allowed: 3 Hours **Maximum Marks: 80**

Note: Attempt any four questions from the given questions. Each question carries equal marks.

Q 1. *i) Define vector space. Show that* $\mathbb{C}$ *is a vector space over* $\mathbb{R}$.

ii) Prove that the union of two subspaces of a vector space V *over a field* F *is a subspace if and only if one of them is a subset of the other.*

Q 2. *i) Let* V *be a vector space over a field* F. *If* S *and* T *are subsets of* V, *prove that*

$$L(S \cup T) = L(S) + L(T).$$

ii) Let V *be vector space of real functions over* $\mathbb{R}$, *show that the set of functions* $S = \{e^{3x},\ x^3,\ x^2\}$ *is L.I.*

Q 3. *i) Define basis of a vector space. Give an example of the following:*

(a) A finite dimensional vector space.

(b) An infinite dimensional vector space.

ii) Prove that any two bases of a finite dimensional vector space have same number of elements.

Q 4. *i) Define dual space. Let* V *be a finite dimensional vector space over a field* F, *dim.*$V = n$ *and* v *is a non-zero vector, then show that there exists* $f \in V^*$ *such that* $f(v) \neq 0$.

ii) Show that the vectors $(1,\ 1,\ 1)$, $(1,\ 0,\ 1)$ *and* $(1,\ -1,\ -1)$ *of* $\mathbb{R}^3$ *form a basis of* $\mathbb{R}^3$ *over* $\mathbb{R}$.

Q 5. *i) If* V *and* W *are two vector spaces over the same field* F. *Show that* $T\colon V \to W$ *is a linear transformation if and only if* $T(\alpha x + \beta y) = \alpha T(x) + \beta T(y)$ *for all* $x,\ y \in V$ *and* $\alpha,\ \beta \in F$.

ii) Show that $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ *defined by* $T(x,\ y) = (x + y,\ x - y,\ y)$ *is a linear transformation. Also find its kernel.*

Q 6. *i) Find the matrix representation of* $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ *defined as*

$$T(x,\ y) = (3x - 4y,\ x + 5y)$$

with respect to the basis $B = \{(1,\ 3),\ (3,\ 4)\}$.

ii) Show that countable union of countable sets is countable. Hence show that $\mathbb{N}\times\mathbb{N}$ *is countable.*

Q 7. *i) Define closed set of* $\mathbb{R}$. *Show that the following sets are closed:*

(a) $\mathbb{N}$

(b) $\mathbb{Z}$

(c) $\mathbb{R}$

ii) Define limit point of a set. Show that 0 *is the only limit point of the set* $\{\frac{1}{n} : n \in \mathbb{N}\}$.

Q 8. *i) Show that every continuous function* $f\colon [a,\ b] \to \mathbb{R}$ *is uniformly continuous.*

ii) Prove that every monotonically increasing sequence $\{a_n\}$ *converges if and only if it is bounded above.*